For the teacher's guide and all answers
please contact info@putterking.com

Future Kings™ Education Program

Subject: Math
Level 3 (Queen)

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Math

Level 3 - Queen

Summary

Area of focus: trigonometry and mathematical proofs

Topics covered:

- Trigonometry
- Right triangle definitions
- Trigonometric functions
- Special right triangles
- Law of sines
- Law of cosines
- Postulates and axioms
- Theorems
- Pythagorean Theorem
- Mathematical proof

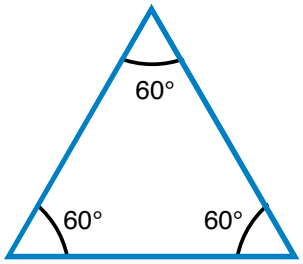
Suggested time to complete (2 hrs):

- Teaching material (40 minutes)
- Practice activity (20 minutes)
- Final project (60 minutes)

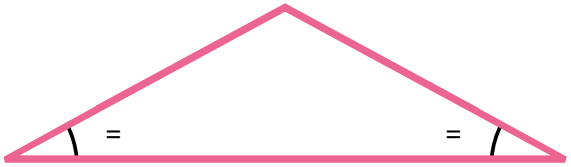
Trigonometry

Trigonometry is the study of the relations of the sides and angles of triangles.

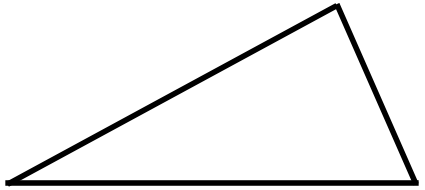
There are three special names given to triangles that tell how many sides (or angles) are equal.



Equilateral Triangle
- Three equal sides
- Three equal angles, always 60°



Isosceles Triangle
- Two equal sides
- Two equal angles

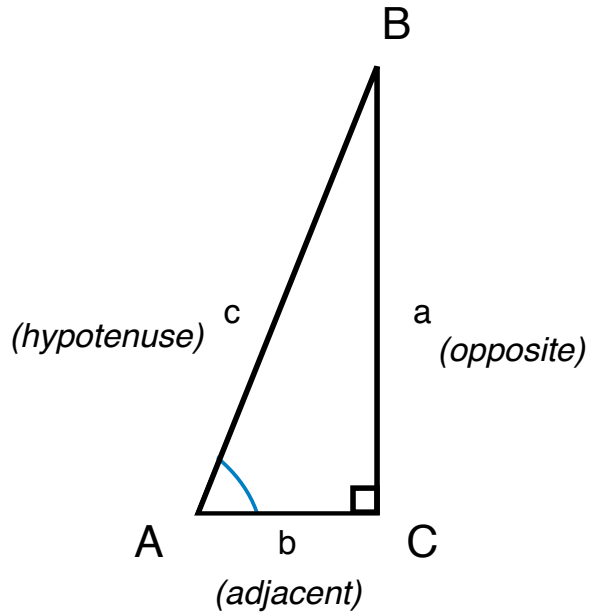


Scalene Triangle
- No equal sides
- No equal angles

Right Triangle Definitions

For any **right triangle** that contains the angle A, the three sides of the triangle are:

- The **hypotenuse** is the side opposite the right angle: **side c**. The hypotenuse is always the longest side of a right triangle.
- The **opposite** side is the side opposite to the angle we are interested in: **side a**.
- The **adjacent** side is the side having both the angle of interest (angle A) and the right angle (angle C): **side b**.



Trigonometric Functions

Trigonometric functions are functions of an angle. They are used to relate the angles of a triangle to the lengths of the sides of a triangle.

Function	Abbreviation	Description
Sine	sin	opposite / hypotenuse
Cosine	cos	adjacent / hypotenuse
Tangent	tan	opposite / adjacent
Cotangent	cot	adjacent / opposite
Secant	sec	hypotenuse / adjacent
Cosecant	csc	hypotenuse / opposite

Trigonometric Functions

There is a useful mnemonic device to help remember the **trigonometric functions**:

SOH-CAH-TOA

Sine = **O**pposite ÷ **H**ypotenuse

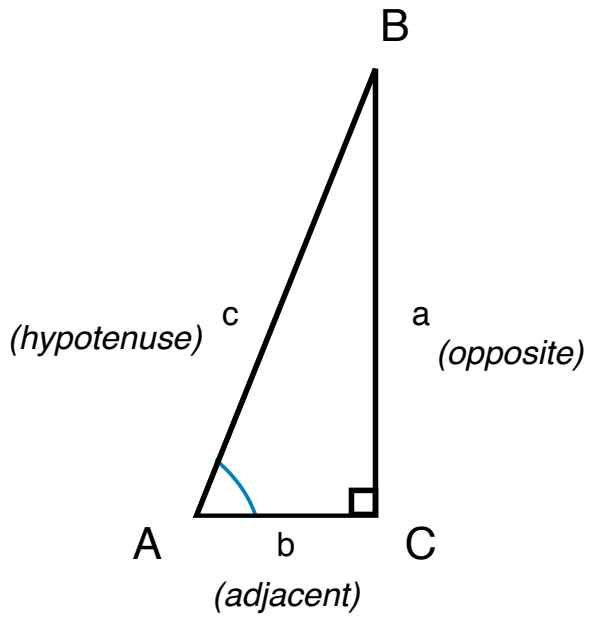
Cosine = **A**djacent ÷ **H**ypotenuse

Tangent = **O**pposite ÷ **A**djacent

$$\sin A = \frac{a}{c}$$

$$\cos A = \frac{b}{c}$$

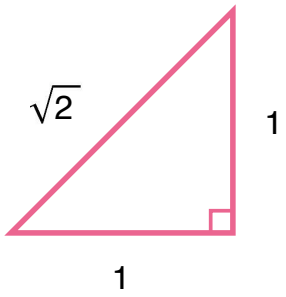
$$\tan A = \frac{a}{b}$$



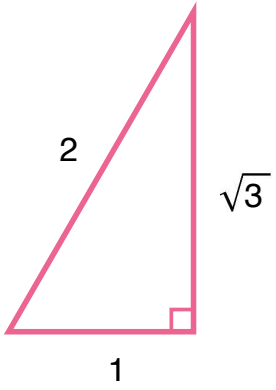
Special Right Triangles

A **special right triangle** is a right triangle whose sides are in a particular ratio which makes calculations on the triangle easier. Knowing the relationships of the angles or ratios of sides of these special right triangles allows one to quickly make calculations.

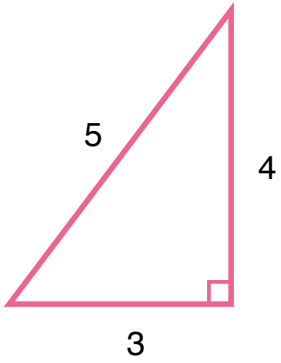
45° - 45° - 90° Triangles



30° - 60° - 90° Triangles



3 - 4 - 5 Triangles

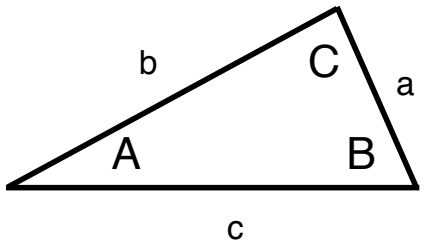


Law of Sines

The **law of sines** is an equation relating the lengths of the sides of a plane triangle to the sines of its angles.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

where *a*, *b*, and *c* are the lengths of the sides of a triangle, and *A*, *B*, and *C* are the opposite angles.

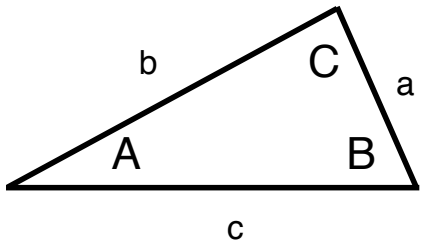


Law of Cosines

The **law of cosines** is an equation relating the lengths of the sides of a plane triangle to the cosine of one of its angles.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

where *a*, *b*, and *c* are the lengths of the sides of a triangle, and *A*, *B*, and *C* are the opposite angles.



Postulates and Axioms

Postulates are statements that are assumed to be true without proof. Postulates are also called **axioms**.

Postulates serve two purposes:

I) to explain undefined terms

II) to serve as a starting point for proving other statements

For present day mathematicians, **axiom** and **postulate** hold a slightly different meaning than they did for the ancient Greeks. In the field of mathematical logic, a clear distinction is made between two notions of axioms: logical (axioms) and non-logical (postulates).

Theorems

Theorems are statements that can be deduced and proved from definitions, postulates, and previously proved theorems.

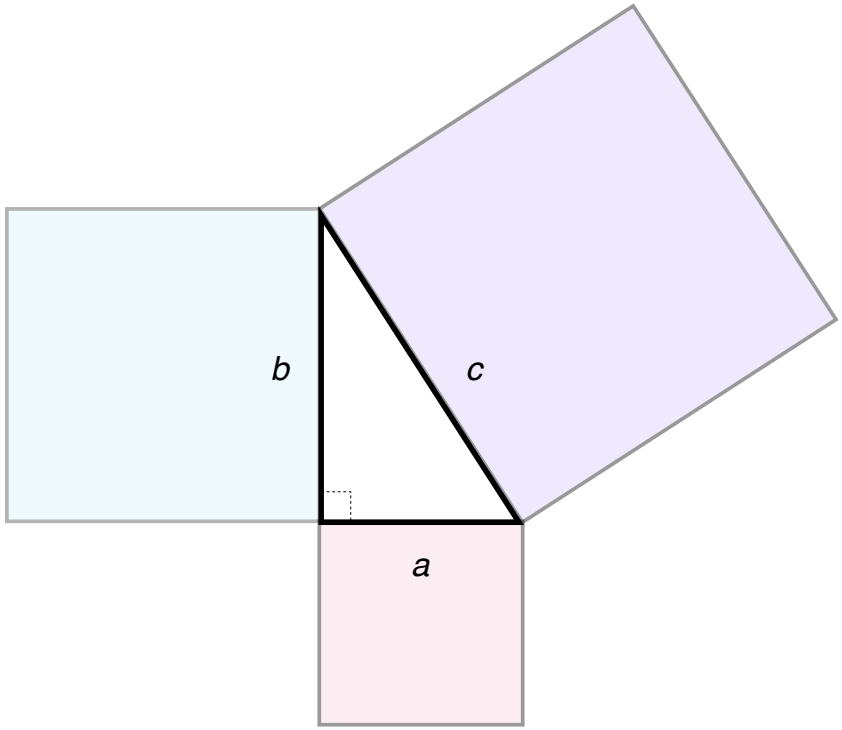
Famous **theorems**:

- Pythagorean Theorem
- Euclid's Proof of the Infinitude of Primes
- The Irrationality of the Square Root of 2
- $\sin^2 \theta + \cos^2 \theta = 1$

Pythagorean Theorem

In any right triangle, the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at a right angle).

$$a^2 + b^2 = c^2$$



Mathematical Proof

A **proof** is a convincing demonstration that some mathematical statement is necessarily true. A **proof** must demonstrate that a statement is true in all cases, without a single exception.

A **proof** is a sequence of steps linked together by modus ponendo ponens (Latin for the way that affirms by affirming). This rule of logic says that if we know that "A implies B", and if we know "A", then we may conclude B.

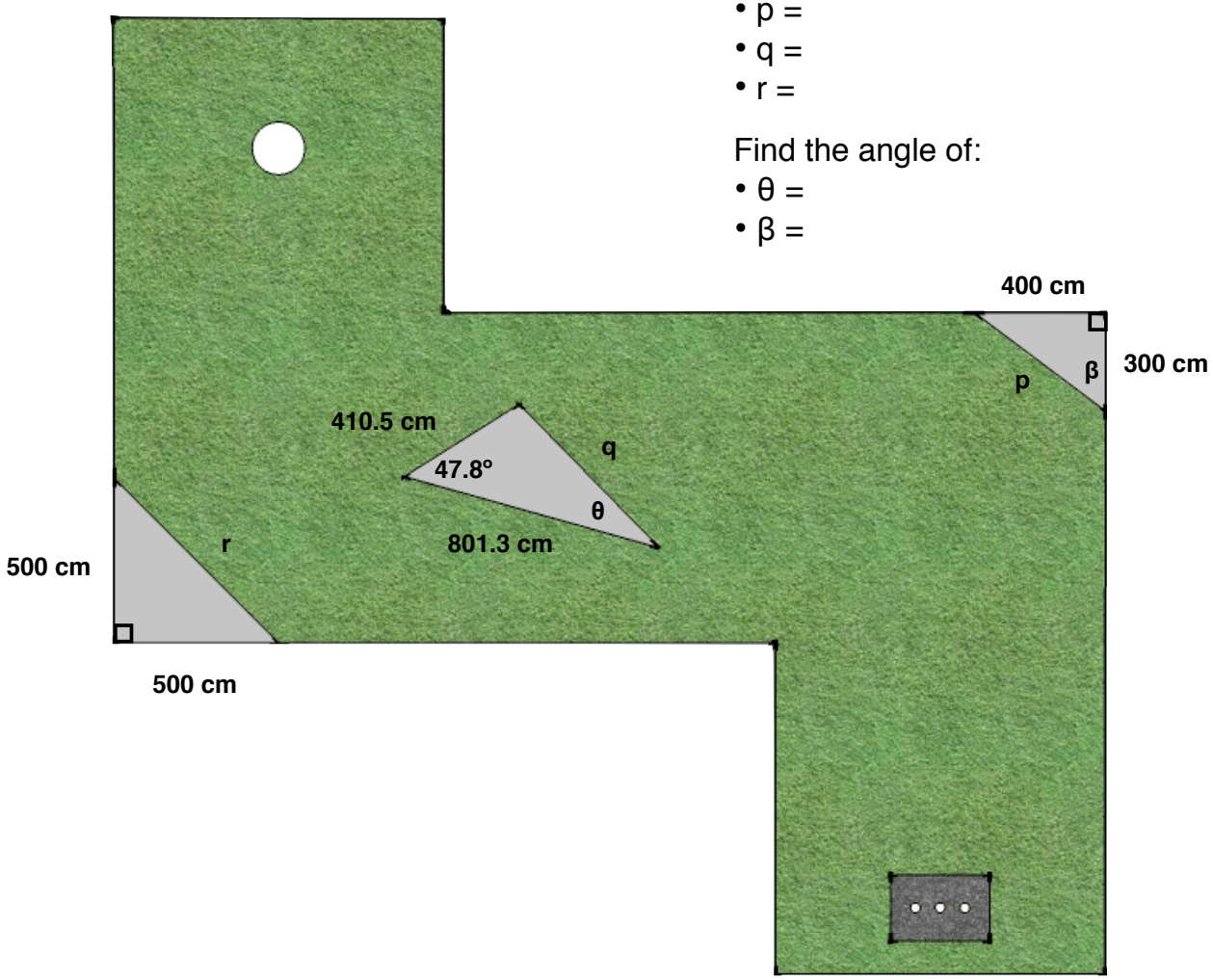
Typical Methods of Proof

- Constructive Proof
- Proof by Contrapositive
- Proof by Contradiction
- Proof by Induction
- Counterexamples

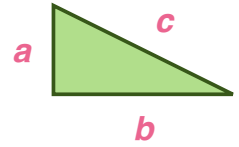
Practice Activity #1

Find the length of:
- p =
- q =
- r =

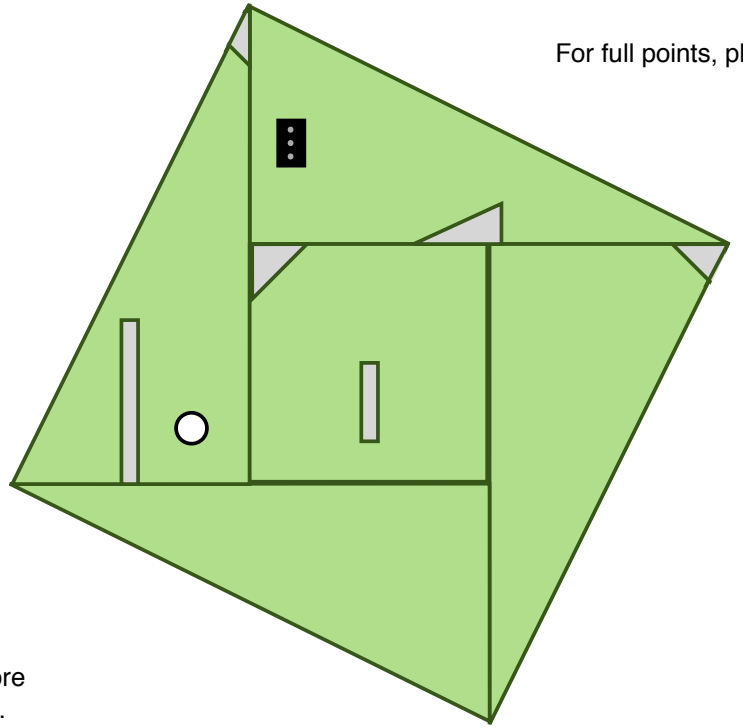
Find the angle of:
- θ =
- β =



Practice Activity #2



Arrange the large green pieces below to prove the
Pythagorean Theorem using the proof by rearrangement method



For full points, please give a written explanation of your proof.

Hint: The miniature golf hole will be more intuitive with the correct rearrangement.

The gray pieces are golf course obstacles.

Final Project

Requirements

Design and label a miniature golf hole that includes at least:

- special right triangles (2)
- equilateral triangle (1)
- isosceles triangle (1)
- scalene triangle (1)

For full points, be sure to:

- use the Pythagorean theorem to find the length of a side of a triangle in your design
- use the law of sines to find the length of a side or the measure of an angle of a triangle in your design
- use the law of cosines to find the length of a side of a triangle in your design
- be creative!

Grading Rubric

	Bogey (70% - 79%)	Par (80% - 89%)	Birdie (90% - 100%)
Triangles	• Student did not include all required triangles • Student incorrectly labeled the triangles	• Student forgot to include a required triangle • Student incorrectly labeled one triangle	• Student included all required triangles • Student correctly labeled all required triangles
Formulas	• Student did not use all of the required formulas to calculate the length of a side or the measure of an angle • Student made multiple calculation errors	• Student forgot to use one of the required formulas • Student made a calculation error	• Student correctly used all required formulas • Student did not make any calculation errors
Design	• Miniature golf hole design is plain and simple • Miniature golf hole design does not exhibit creativity	• Miniature golf hole design is playable but rather plain	• Miniature golf hole design is playable and aesthetically pleasing • Miniature golf hole design is creative and original