Rockets

Momentum is the product of the mass and velocity of an object.

$$L = mV$$

<u>Newton's Law</u>

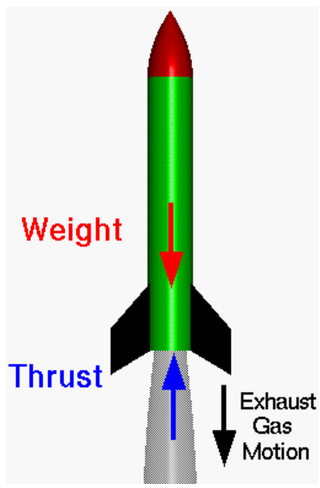
Force on an object is equal to the rate of change of momentum.

$$T = \frac{\Delta L}{\Delta t}$$

Two possibilities:

$$\Delta L = m\Delta V$$

$$\Delta L = V\Delta m$$



The force on the rocket is called "thrust."

$$T = \frac{V\Delta m}{\Delta t} = V\frac{\Delta m}{\Delta t}$$

Let us write this equation in two steps

$$Q = \frac{\Delta m}{\Delta t}$$

$$T = VQ$$

For a rocket, Q, the mass flow-rate = propellant mass divided by the "burn time."
For a rocket, V = exhaust velocity

For a rocket, thrust is important, but what is more important is the quantity called **impulse**.

For a rocket, impulse I = thrust multiplied by burn time.

You can see why impulse is more important than thrust itself. Imagine yourself pushing a chair for 5 seconds. Now imagine yourself pushing a chair with the same force for 15 seconds. **Impulse is 3 times more in your second push and the chair goes farther.**

$$\mathbf{I} = \mathbf{T}\Delta\mathbf{t}$$

For a rocket, total mass at launch = A = payload (astronaut + his/her instruments + capsule) + propellant mass + propellant casing
For a rocket, total mass as burnout = B = payload (astronaut + his/her instruments + capsule) + propellant casing
Sometimes, the propellant casing is also ejected. Depending on whether your rocket is ejecting the propellant casing, you have to change your calculation for B.

Velocity at Launch = zero (rocket is sitting on the launch pad)
Velocity at burn time = U
Ideal Rocket Equation:

$$\mathbf{U} = \mathbf{V}[\ln(\mathbf{A}) - \ln(\mathbf{B})]$$

Caution: Effect of gravity and aerodynamic drag on the rocket are ignored in this equation.

Step-by-Step Calculation
You can find model rocket specifications at:
http://www.apogeerockets.com/

We picked a rocket with the following specifications:
Propellant Mass = 40.7 g = 0.0407 kg
Initial Mass of Rocket = 71 g = 0.071 kg
Mass of rocket casing = 71 g – 40.7 g = 30.3 g = 0.0303 kg
Burn Time = 7.8 s
Total Impulse = 80 N.s

We chose a payload = 10 g = 0.01 kg

Now, let us calculate the final velocity of our rocket at burn-out.

Q = Propellant Mass divided by burn-out time = 0.0407 / 7.8 = 0.005218 kg/s
T = Total Impulse divided by burn-out time = 80 / 7.8 = 10.26 N
V = Thrust (T) divided by mass flow-rate (Q) = 10.26 / 0.005218 = 1966 m/s

A = Initial Mass = Mass of Rocket + Payload = 0.071 + 0.01 = 0.081 kg
B = Final Mass = Rocket Casing + Payload = 0.0303 + 0.01 = 0.0403 kg

Final velocity = V [$\ln(A) - \ln(B)$] = 1966 [$\ln(0.081) - \ln(0.0403)$] = 1372 m/s

The actual final velocity will be much less than what we just calculated, because the rocket equation ignores the effect of gravity and aerodynamic drag force on the rocket.

Some standard velocities for a rocket launch

When a rocket is fired straight up, to escape from earth's gravity, U = 11,200 m/s
When a rocket is fired eastward from a point on the equator, to escape from earth's gravity, U = 10,745 m/s
When a rocket is fired westward from a point on the equator, to escape from earth's gravity, U = 11,655 m/s

Explain the difference between eastward and westward fires.

Cape Canaveral; 55 miles east of Orlando, Florida.

Low Earth Orbit (LEO)

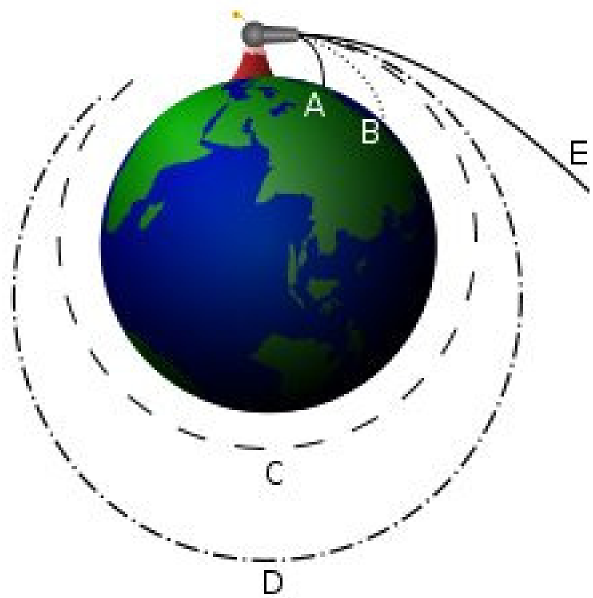
The International Space Station is in a LEO that varies from 320 km (199 mi) to 400 km (249 mi) above the Earth's surface.

To put a satellite in LEO, U = 7,800 m/s.

You cannot fire a rocket at this high value of U. It will get very hot due to friction with air and will turn to ashes; just like a shooting star.

We reach this high value of U, in three steps by using a three-stage rocket.

Various Orbits



1. If the speed (U) is low, it will simply fall back on Earth. (A and B)
2. If the speed (U) is the orbital velocity at that altitude it will go on circling around the Earth along a fixed circular orbit just like the moon. (C)
3. If the speed (U) is higher than the orbital velocity, but not high enough to leave Earth altogether (lower than the escape velocity) it will continue revolving around Earth along an elliptical orbit. (D)
4. If the speed (U) is very high, it will indeed leave Earth. (E)